

Summary of Working Group 1: Theory Part

Osamu Yasuda

*Department of Physics, Tokyo Metropolitan University
1-1 Minami-Osawa Hachioji, Tokyo 192-0397, Japan*

Abstract

I will summarize theoretical issues in Working Group 1 at Nufact'01. In particular I will clarify a controversy on the optimum baseline L muon energy E_μ for measurements of the CP phase at a neutrino factory. If the uncertainty of the matter effect is assumed to be $\pm 10\%$, which may be slightly too pessimistic, then the results from two groups agree ($L \sim 1000\text{km}$, $E_\mu \sim 10\text{GeV}$).

1 Model Building

In the three flavor framework of neutrino oscillations, the only two parameters whose lower bound is not known in the mixing matrix U_{MNSP} ¹ are θ_{13} and δ , and the magnitude of $|U_{e3}| = s_{13}$ is expected to give us a clue to model buildings. Tanimoto [2] gave various examples of models which predict the magnitude of U_{e3} . Some (e.g., anarchy models) predict large $U_{e3} \sim O(\lambda)$ where $\lambda \simeq 0.2$, other models (those with some flavor symmetry or with the conformal field fixed point) predict $U_{e3} \sim O(\lambda^2 - \lambda)$ or $O(\lambda^3 - \lambda)$, while Zee type models typically give tiny $U_{e3} \ll \lambda^3$. Some GUT models predict testable sum rules among V_{CKM} and U_{MNSP} . Koide [5] emphasized that the deviation of the solar mixing angle from the maximal mixing is small in the Zee model: $1 - \sin^2 2\theta_\odot \leq (1/16)(\Delta m_\odot^2 / \Delta m_{\text{atm}}^2)^2$. This relation can be checked experimentally in the near future. Kitabayashi [6] presented a model with a minimal $SU(3)_L \times U(1)_N$ gauge symmetry, an approximate $L_e - L_\mu - L_\tau$ symmetry and a discrete Z_4 symmetry. This model yields the LMA MSW solution and explains the hierarchy $|\Delta m_\odot^2| \ll |\Delta m_{\text{atm}}^2|$ by loop corrections. Leung [7] considered effective operators relevant for generating small Majorana neutrino

Email address: yasuda@phys.metro-u.ac.jp (Osamu Yasuda).

¹ Petcov [1] proposed that the neutrino mixing matrix be referred to as the Pontecorvo-Maki-Nakagawa-Sakata [3,4] matrix. Here I refer to the matrix as U_{MNSP} .

masses. All the effective higher dimensional ($5 \leq d \leq 11$) operators are compiled which conserve baryon number and the SM gauge invariance but violate lepton number by two units. If neutrinoless double β decays are observed, then a number of operators will be reduced. Geer [8] discussed how precise measurements of the oscillation parameters would give us useful information at high energy scale, by taking a specific SUSY SO(10) GUT model with the LMA MSW solution. In this particular example, superbeam experiments and/or neutrino factories will constrain the possible values of the GUT-scale parameters.

2 Analysis of solar and atmospheric neutrinos and $(\beta\beta)_{0\nu}$

Lisi [9] reported on analysis of the atmospheric neutrino data in two, three and four flavor frameworks. In the two flavor framework it was shown that $\nu_\mu \leftrightarrow \nu_\tau$ oscillation with maximal mixing is robust. Also in the three flavor scenario, by combining with the CHOOZ data the dominant oscillation in the atmospheric neutrino was shown to be $\nu_\mu \leftrightarrow \nu_\tau$, possibly with small contribution of $\nu_\mu \leftrightarrow \nu_e$. It was pointed out that non-hierarchical scenario where $\Delta m_\odot^2 \sim \Delta m_{\text{atm}}^2$ holds are not totally excluded yet. In the four flavor case the data were analyzed with the so-called (2+2)-scheme, and some contribution of $\nu_\mu \leftrightarrow \nu_s$ is still allowed at present.

Peña-Garay [10] presented analysis of the solar neutrino data in two, three and four flavor mixings. In the case of two flavor active oscillations, the LMA MSW solution is the best fit, and it is followed by the LOW solution. The SMA MSW and vacuum solutions seem to be disfavored now. In the two flavor sterile oscillations, the SMA MSW solution is the best fit. In the three flavor scenario, with the CHOOZ constraint, the dominant channel is $\nu_e \leftrightarrow (\nu_\mu - \nu_\tau)/\sqrt{2}$, i.e., $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\tau$ oscillations occur with equal weight. In the four neutrino mixing with the (2+2)-scheme, the oscillation probability is characterized by one additional parameter $|U_{s1}|^2 + |U_{s2}|^2$ which becomes 0 (1) for pure active (sterile) oscillation, respectively. By combining the analyses of the solar and atmospheric neutrino data, it was found before the SNO data that the case with pure $\nu_e \leftrightarrow \nu_s$ in the solar neutrinos and pure $\nu_\mu \leftrightarrow \nu_\tau$ in the atmospheric neutrinos is close to the best fit. Even after the SNO data this solution is not excluded yet [11].

Petcov [1] gave a talk on the neutrino mass spectrum and CP violation in the lepton sector in the three flavor framework. Neutrinoless double β decay experiments in the future can provide information on the lightest neutrino mass and the CP violation in the lepton sector. In particular, if a positive result ($m_{\nu_e} \geq 0.35\text{eV}$) is reported from neutrinoless double β decay experiments, then combining the data of ^3H β decay experiments of the KATRIN project,

it would allow us to determine m_1, m_2, m_3 and to know the existence of the CP violation in the lepton sector.

3 Various issues in neutrino factory and superbeams

In the past there have been a lot of discussions on the physics potential of neutrino factories and conventional superbeams, and people investigated the optimum baseline and the muon (or neutrino) energy to determine θ_{13} , the sign of Δm_{32}^2 and the CP phase δ of the MNSP matrix in the three flavor framework. Recent progress includes the effects of correlations of errors of the oscillation parameters, the background effects, etc. The ultimate purpose of neutrino factories and conventional superbeams is to determine δ . So far all the discussions are based on indirect measurements of CP violation, i.e., assuming the three flavor mixing, a quantity

$$\Delta\chi_{\text{indirect}}^2 = \sum_j \frac{[N_j(\delta) - N_j(\delta = 0)]^2}{\sigma_j^2}, \quad (1)$$

which compares the difference of the cases of $\delta \neq 0$ and $\delta = 0$ with the errors, is optimized. In (1) $N_j(\delta)$ stands for a binned number of events with δ and it is understood that $\Delta\chi_{\text{indirect}}^2$ is optimized with respect to other parameters in $N_j(\delta = 0)$. As has been criticized by some people [12,13], this approach does not directly deal with CP violating processes, since ν and $\bar{\nu}$ are not directly compared and what we wish to have is an analogue of $N(K_L \rightarrow 2\pi)/N(K_L \rightarrow 3\pi)$ in the $K^0 - \bar{K}^0$ system [14]. However, such quantity has not been discovered yet in the case of neutrino factories or any very long baseline experiments, since the dependence of the probabilities for ν and $\bar{\nu}$ on δ and A is such that $P(\nu_\mu \rightarrow \nu_e) = f(E, L; \theta_{ij}, \Delta m_{ij}^2, \delta; A)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = f(E, L; \theta_{ij}, \Delta m_{ij}^2, -\delta; -A)$, where f is a certain function and $A \equiv \sqrt{2}G_F N_e$ stands for the matter effect, and the quantity obtained from experiments on $\bar{\nu}$ is $f(\cdots, -\delta; -A)$, so that comparison between $f(\cdots, \delta; A)$ and $f(\cdots, -\delta; A)$ is impossible in a strict sense. Therefore all the discussions below basically use indirect measurements of CP violation.

One of the important issues in the discussions on uncertainties of the parameters is that of the density of the Earth. Geller, a geophysicist working on the three dimensional structure of the Earth, explained the density distribution in the Earth's Interior to physicists [15]. To determine the density, there are complications. The neutrino path goes through the upper mantle as well as the crust, so one needs both crust models and regional upper mantle models which are not well established with great accuracy at present. The distribution of the density is much less well constrained than seismic velocities which

are known relatively well. Despite all these problems, from ballpark guesses rather than rigorous error estimation procedures, he concluded that the accuracy of estimates of the average density along the neutrino beam is at worst $\pm 10\%$ and is probably within $\pm 5\%$. There were two talks in which a method was presented to treat the density profile which is in general not constant. Takasugi [16] discussed T violation and derived a formula up to third order in $\Delta m_{21}^2 L/2E$ and $\delta a(x)L/2E$ where $\delta a(x)$ stands for deviation from the PREM. He concluded that the asymmetric matter contribution to T violation is negligible. Ota [17] presented a method of Fourier expansion of the density profile to discuss the matter effect and its ambiguity, and showed that only the first few coefficients are important.

Lindner [18] discussed the optimization of sensitivity to $(\Delta m_{32}^2, \sin^2 2\theta_{23})$, $\sin^2 2\theta_{13}$, $(\Delta m_{21}^2, \sin^2 2\theta_{12})$, and δ , taking into account the correlations of errors of the oscillations parameters and assuming that background effects are negligible. In the case of $(\Delta m_{32}^2, \sin^2 2\theta_{23})$, the optimum baseline lies between 1000km and 5000km. In the case of $\sin^2 2\theta_{13}$, $1000\text{km} \lesssim L \lesssim 5000\text{km}$ gives similar results for $\sin^2 2\theta_{13} \gtrsim 10^{-2}$ and the optimum baseline is between 7000km and 8000km for $\sin^2 2\theta_{13} \lesssim 10^{-3}$. Sensitivity to $(\Delta m_{21}^2, \sin^2 2\theta_{12})$ turns out to be much worse than that of the KamLAND experiment [19]. As for δ , the optimum baseline is around 3000km and high energy ($E_\mu \gtrsim 40\text{GeV}$) is preferred. This result agrees with [20] and [21].

Burguet-Castell and Mena [22] refined their analysis in [20] of the sensitivity to δ and θ_{13} by considering the full range of these parameters. They found that there is twofold degeneracy in (θ_{13}, δ) . This is because the expansion of the oscillation probabilities to second order in $|\theta_{13}| \ll 1$ gives a quadratic equation in θ_{13} :

$$P_{\nu_e \nu_\mu(\bar{\nu}_e \bar{\nu}_\mu)} = X_\pm \theta_{13}^2 + Y_\pm \theta_{13} \cos(\delta - \frac{\Delta_{13} L}{2}) + P^{sol},$$

where notations are given in [23]. To resolve this degeneracy, a combination of two baselines seems necessary.

Pinney [21] also discussed the optimum baseline and the muon energy of a neutrino factory for δ , by taking into account the systematic errors and the correlations of errors of the oscillation parameters as well as the density of the Earth. If the background fraction f_B is 10^{-5} and the uncertainty of the matter effect is $\pm 5\%$, then $E_\mu \sim 50\text{GeV}$ and $L \sim 3000\text{km}$ is the optimum parameter set (this result agrees with those in [20] and [18]), but if $f_B = 10^{-3}$ or if the uncertainty of the matter effect is $\pm 10\%$, then the lower muon energy ($E_\mu \lesssim 20\text{GeV}$) and the shorter baseline ($L \sim 1000\text{km}-3000\text{km}$) gives the optimum. In [24] it was shown that $\Delta \chi_{\text{indirect}}^2$ used in the analysis in [24] has behavior

$$\Delta\chi_{\text{indirect}}^2 \sim \left(\frac{J}{\sin\delta}\right)^2 \frac{1}{E_\mu} \left(\sin\delta + \text{const} \frac{\Delta m_{32}^2 L}{E_\mu} \cos\delta\right)^2 \quad (2)$$

for large E_μ (J stands for the Jarlskog parameter), so that sensitivity to CP violation is lost in the large E_μ limit. (2) also suggests that neutrino factories with $E_\mu \sim 50\text{GeV}$ and $L \sim 3000\text{km}$ are mainly sensitive to $\sin\delta$ instead of $\cos\delta$.²

Koike [26] presented their discussions in [25] on the optimum baseline and the muon energy at neutrino factories. His main strategy is to use

$$\Delta\tilde{\chi}_{\text{indirect}}^2 = \sum_j \frac{[N_j(\delta) - \bar{N}_j(\delta) - N_j(\delta=0) + \bar{N}_j(\delta=0)]^2}{\sigma_j^2}, \quad (3)$$

where $\bar{N}_j(\delta)$ stands for a suitably normalized binned number of events for $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillations and again optimization with respect to other parameters in $N_j(\delta=0)$ and $\bar{N}_j(\delta=0)$ is understood. He claimed that $\Delta\tilde{\chi}_{\text{indirect}}^2$ ($=\chi_2^2(\delta_0)$ in the notation of [25]) is sensitive to $\sin\delta$ whereas $\Delta\chi_{\text{indirect}}^2$ in (1) is mainly to $\cos\delta$. In my opinion, however, $\Delta\chi_{\text{indirect}}^2$ in (1) and $\Delta\tilde{\chi}_{\text{indirect}}^2$ in (3) are basically the same, as their large E_μ behaviors are the same after the correlations of errors are taken into account and both use indirect measurements of CP violation. In fact, the conclusion by Koike that $E_\mu \sim 10\text{GeV}$ and $L \sim 1000\text{km}$ optimizes the sensitivity to δ almost agrees with that in [24] where the case of $\Delta C \equiv$ the uncertainty of the matter effect $=\pm 10\%$ and $f_B = 10^{-5}$ was also considered (See Fig. 8 in [24]). Notice that Koike assumed that all the uncertainty of the oscillation parameters as well as the matter effect is $\pm 10\%$.

Wang [27] also discussed the optimum baseline for experiments with conventional superbeams and for neutrino factories by introducing a figure of merit which is defined as

$$F_M = \frac{P}{\sqrt{(P + f_B)/N + (gP)^2 + (rP)^2}},$$

where P is the oscillation probability, N is the number of events, f_B is the background fraction, r is the systematic uncertainty, and g is the uncertainty of the flux and the cross sections. He concluded that $L \simeq 2000\text{km} - 3000\text{km}$ is preferred for measurements of $\sin^2 2\theta_{13}$, δ and the sign of Δm_{32}^2 .

² [25] criticized the analysis with $\Delta\chi_{\text{indirect}}^2$ ($=\chi_1^2(\delta_0)$ in the notation of [25]) by saying that $\chi_1^2(\delta_0)$ has the behavior $\chi_1^2(\delta_0) \sim E_\mu (J/\sin\delta)^2 (\cos\delta \pm 1)^2$ in the large E_μ limit and has sensitivity mainly to $\cos\delta$. However, this argument does not include the effects due to the correlations of errors, and once optimization with respect to other parameters is done, the correct behavior (2) is obtained.

Minakata [14] discussed the physics potential of the phase II JHF experiment which is expected to have 4MW power and 1Mt water Cherenkov detectors. He examined if it is possible to determine the $\sin^2 2\theta_{13}$, the sign of Δm_{32}^2 and δ by introducing a CP trajectory diagram, in which a point sweeps out an ellipse in the $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ plane as δ ranges from 0 to 2π . The behavior of the ellipse varies depending on the sign of Δm_{32}^2 , and in certain cases it is possible to determine δ and Δm_{32}^2 in the phase II JHF experiment. On the other hand, if the oscillation parameters lie in unlucky regions, then there is a twofold ambiguity in $(\delta, \Delta m_{32}^2)$. This ambiguity is resolved by putting another detector at $L=700\text{km}$.

Okamura [28] discussed the physics potential of a very long baseline experiment ($L=2100\text{km}$) with conventional superbeams ($3\text{GeV} \lesssim E_{\text{proton}} \lesssim 6\text{GeV}$). Taking into account the correlations of errors and the background effects, he examined sensitivity to the sign of Δm_{32}^2 , $\sin^2 2\theta_{13}$, $(\Delta m_{21}^2, \sin^2 2\theta_{12})$, and δ . With such a long baseline, sensitivity to the sign of Δm_{32}^2 is very good, but sensitivity to $(\Delta m_{21}^2, \sin^2 2\theta_{12})$ is poor as is naively expected. Sensitivity to δ is good for $\sin^2 2\theta_{13} \gtrsim 0.05$, assuming the detector size $100\text{kt}\cdot\text{yr}$.

4 Four neutrino scenarios

To account for the solar and atmospheric neutrino data as well as the LSND result in terms of neutrino oscillations, one has to have at least four neutrino mass eigenstates since one would need at least three independent mass squared differences. Four neutrino scenarios are classified into two categories, the (2+2)-scheme and the (3+1)-scheme, depending on the number of degenerate mass eigenstates separated by Δm_{LSND}^2 .

Peres [29] reviewed the phenomenology of the (3+1)-scheme, which is excluded by the LSND data at 95%CL but allowed at 99%CL. The upper left 3×3 components of the MNSP matrix for the (3+1)-scheme are supposed to be close to those for the three flavor case, so it gives a good fit to the data of the solar and atmospheric neutrino data. The (3+1)-scheme is allowed only for $\Delta m_{\text{LSND}}^2 \simeq 0.9, 1.7, 6.0 \text{ eV}^2$, so the upper limit on the absolute neutrino masses would constrain the allowed region of this scheme. β decay experiments, neutrinoless double β decay experiments and supernova neutrinos give some bounds.

Bell [30] discussed how relic neutrino asymmetries may be generated in the early universe via active-sterile oscillations. In the two flavor framework it is known that asymmetry between ν and $\bar{\nu}$ is generated through active-sterile oscillations under a certain condition for neutrino masses and the mixing parameters. The implications of this mechanism to Big Bang Nucleosynthesis

was given in the case of a particular (2+2)-scheme model where a $\nu_\mu - \nu_\tau$ pair is separated by a $\nu_e - \nu_s$ pair, and the conclusion was that $\delta N_\nu^{\text{eff}} \simeq -0.3$ (+0.1) for positive (negative) asymmetry, respectively.

Donini [31] compared the physics reach of a neutrino factory in the (3+1)- and (2+2)-schemes. In both schemes huge CP violating effects can be observed with a 1Kt detector, $2 \times 10^{-20} \mu$'s and $L = 10 - 100 \text{ km}$ in the $\nu_\mu \rightarrow \nu_\tau$ channel. In case a conclusive confirmation of the LSND result is absent, it is difficult to discriminate the (3+1)-scheme from the three flavor case unless the mixing angle is large and it is easy to discriminate the (2+2)-scheme from the three flavor case.

References

- [1] S. Petcov, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_petcov.pdf].
- [2] M. Tanimoto, in these proceedings [arXiv:hep-ph/0106064].
- [3] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [4] B. Pontecorvo, Sov. Phys. JETP **26** (1968) 984 [Zh. Eksp. Teor. Fiz. **53** (1968) 1717].
- [5] Y. Koide, in these proceedings [arXiv:hep-ph/0109157].
- [6] T. Kitabayashi and M. Yasue, in these proceedings [arXiv:hep-ph/0106144].
- [7] C. N. Leung, in these proceedings [arXiv:hep-ph/0108130].
- [8] S. Geer, in these proceedings
[http://www-prism.kek.jp/nufact01/May26/WG1/26wg1_geer.pdf].
- [9] E. Lisi, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_lisi.pdf].
- [10] C. Pena-Garay, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_garay.pdf].
- [11] M. C. Gonzalez-Garcia, M. Maltoni and C. Pena-Garay, hep-ph/0108073.
- [12] J. Sato, Nucl. Instrum. Meth. A **472** (2000) 434 [arXiv:hep-ph/0008056].
- [13] A. Konaka, private communication.
- [14] H. Minakata and H. Nunokawa, arXiv:hep-ph/0111130.
- [15] R. Geller, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_geller.pdf].

- [16] E. Takasugi, in these proceedings
[http://www-prism.kek.jp/nufact01/May29/WG1/29wg1_takasugi.pdf].
- [17] T. Ota, in these proceedings
[http://www-prism.kek.jp/nufact01/May29/WG1/29wg1_ota.pdf].
- [18] M. Lindner, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_lindner.pdf].
- [19] K. Ishihara, in these proceedings
[http://www-prism.kek.jp/nufact01/May26/WG1/26wg1_ishihara.pdf].
- [20] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. B **579** (2000) 17 [Erratum-ibid. B **593** (2000) 731] [[arXiv:hep-ph/0002108](https://arxiv.org/abs/hep-ph/0002108)].
- [21] J. Pinney, in these proceedings [[arXiv:hep-ph/0106210](https://arxiv.org/abs/hep-ph/0106210)].
- [22] J. Burguet-Castell and O. Mena, in these proceedings [[arXiv:hep-ph/0108109](https://arxiv.org/abs/hep-ph/0108109)].
- [23] J. Burguet-Castell, M. B. Gavela, J. J. Gomez-Cadenas, P. Hernandez and O. Mena, Nucl. Phys. B **608** (2001) 301 [[arXiv:hep-ph/0103258](https://arxiv.org/abs/hep-ph/0103258)].
- [24] J. Pinney and O. Yasuda, Phys. Rev. D **64** (2001) 093008 [[arXiv:hep-ph/0105087](https://arxiv.org/abs/hep-ph/0105087)].
- [25] M. Koike, T. Ota and J. Sato, [arXiv:hep-ph/0011387](https://arxiv.org/abs/hep-ph/0011387).
- [26] M. Koike, in these proceedings
[http://www-prism.kek.jp/nufact01/May29/WG1/29wg1_koike.pdf].
- [27] Y. F. Wang, in these proceedings
[http://www-prism.kek.jp/nufact01/May28/WG1/28wg1_wang.pdf].
- [28] N. Okamura, in these proceedings
[http://www-prism.kek.jp/nufact01/May29/WG1/29wg1_okamura.pdf].
- [29] O. L. Peres, in these proceedings
[http://www-prism.kek.jp/nufact01/May29/WG1/29wg1_peres.pdf].
- [30] N. F. Bell, in these proceedings [[arXiv:hep-ph/0108123](https://arxiv.org/abs/hep-ph/0108123)].
- [31] A. Donini and D. Meloni, in these proceedings [[arXiv:hep-ph/0107274](https://arxiv.org/abs/hep-ph/0107274)].